Derivatives Analysis and Valuation

Derivatives is a financial instrument which derives value from an underlying asset i.e shares, commodity
Example: Value of option changes with respect to changes in price

OPTIONS: Option is a financial contract that gives holder right but not obligation to buy or sell an underlying asset at a predetermined price (STRIKE PRICE) on or before expiry date
There are two types of Options:

- Call Option: Gives holder right to buy underlying asset at strike price.
  (Exercised when price at Maturity > Exercise price)
- Put Option: Gives holder right to sell underlying asset at strike price
  (Exercised when price at Maturity < Exercise price)

For Eg: Rohan expects price of Reliance share to go up to 600 in a month's time. CMP is 500. Rohan Buys call option at a strike price of 510, which gives him right to purchase this share at 510 after one month
CASE 1: Actual price after one month is 580. Rohan will exercise call option and purchase share at 510 and then sell at 580. Profit = 580-510= 70 per share
CASE 2: Actual price falls after one month is 450. Rohan will do nothing and call option will lapse

Important Point: Rohan will have to pay an amount in advance for entering into option contract
This amount is technically called as option premium

In our example, Let's say Rohan bought call option at 510 and paid premium of 10 at beginning to seller of call option.
In case 1: Net profit will be 70-10= Rs 60
In case 2: Net Loss will be Rs 10

There are 2 Parties in option contract: 1) Option Buyer and 2) Option seller/ writer
Option buyer pays premium to option seller for purchase of option

Note: Loss to option buyer is limited to option premium while Loss to option seller is unlimited

There are 2 types of options:

- AMERICAN OPTION: Can be exercised anytime on or before maturity
- EUROPEAN OPTION: Can be exercised only on maturity

Note: Option Premium under American option > European option

	CALL OPTION	PUT OPTION
In the Money (ITM) option	EP < CMP	EP > CMP
Out of the Money (OTM) option	EP > CMP	EP < CMP
At the Money (ATM) option	EP = CMP	EP = CMP

Intrinsic value of Option and Time Value of Option
Option Premium has 2 parts: 1) Intrinsic value and 2) Time Value
For Example: Call option exercise price is 500, Time to maturity is 3 months. CMP is 510 and Option premium is 18.

Option Premium of 18 is comprised of 2 things:
Intrinsic value (510 - 500) = 10 and Time Value of money (18-10) = 8
Intrinsic value in layman language means in case price after 3 months is 510, then value of option will be difference between CMP-EP i.e. 510-500 = 10

Option Premium reduces as it approaches towards maturity due to time value of money. As option reaches maturity, time value of money goes down
Expected Value of Option if multiple maturity price is given = (Total Profit on maturity x Probability)

There are 3 primary participants in Derivative Market

PARTICIPANT	OBJECTIVE	EXAMPLE
Hedgers	To reduce risk	Farmers entering into future contracts
Speculators	Purely speculation purpose to earn profit	Buying of call option in expectation of increase in price
Arbitrageur	Exploit price differences	Buying at low price from NSE, selling it at higher price in BSE

OPTIONS VALUATION: Basically, here we calculate value of option (Fair value of option premium) and then we will compare it with actual premium and accordingly we will decide whether to buy option or not

If Fair value of option > Actual Premium [Buy Option]

If Fair value of option < Actual Premium [Not to Buy Option]

Valuation methods:

1. Binomial Method and
2. 2. Black Scholes Method

Binomial method: This method assumes that there are only 2 possible outcomes of market prices on maturity. One is more than CMP and other lower than CMP

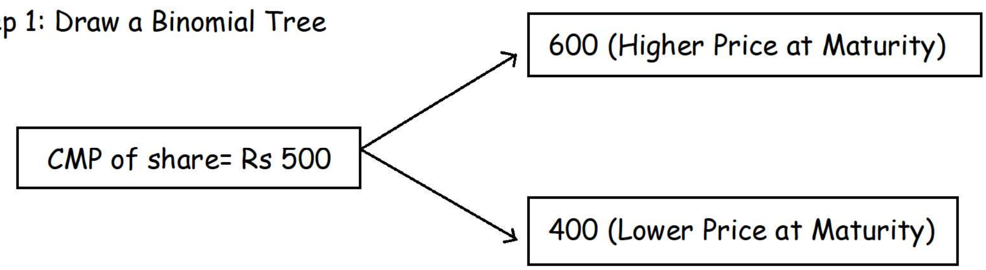
Two approaches to calculate value of option as per Binomial method is:

1. Risk Neutral Probability method
2. Delta Hedging

Risk Neutral Probability Method

Single Period Binomial Method: Applied generally in case of European options where option can be exercised only on maturity Accordingly on maturity, market price can either be higher than CMP or lower than CMP

Step 1: Draw a Binomial Tree



Step 2: Calculate Risk Neutral Probability: $P = \frac{\left(1+\frac{R}{100}\right) - d}{\mu - d}$ or $\frac{e^{rt} - d}{\mu - d}$

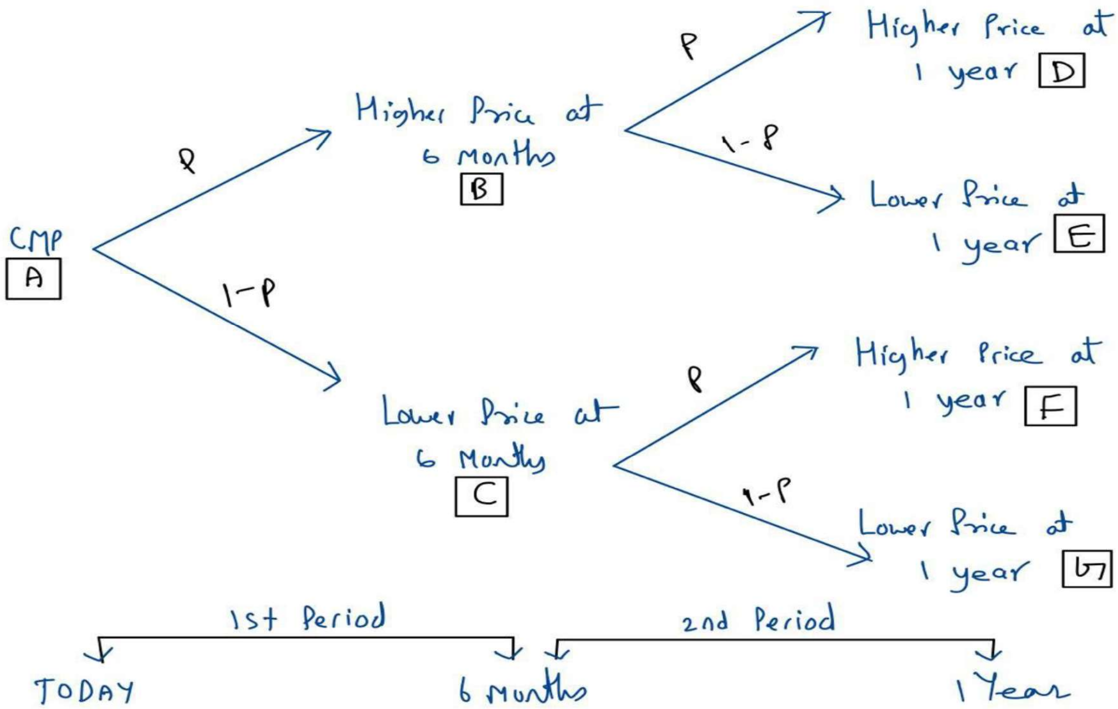
$P =$ Probability of price moving higher
$R =$ Risk Free Return
$\mu = \frac{\text{Higher Price at Maturity}}{\text{CMP}}$
$d = \frac{\text{Lower Price at Maturity}}{\text{CMP}}$

Step 3: Value of Option = $\frac{[\text{Pay off if price Increases} \times P] + [\text{Pay off if price decreases} \times (1-P)]}{1+\frac{R}{100}}$

Two Period Binomial Method: Applied in case of American option where options can be exercised before maturity as well. For Example: Maturity of call option is 1 year. And Investor wants to know value of option if he expects to exercise option either at 6 months or on maturity.

In above case, 2 period binomial tree will be prepared.

Step 1: Draw a Binomial Tree



Step 2: Calculate probability as discussed in single binomial tree model

Note: Probability will be same for 1st period and 2nd period because percentage increase and decrease in prices are same

Step 3: Calculate value of option at Point B and C respectively, by using below formula=

$$\frac{[\text{Pay off if price Increases } \times \text{P}] + [\text{Pay off if price decreases } \times (1-\text{P})]}{1+\frac{\text{R}}{100}}$$

Step 4: By using value of option at Point B and C, calculate value of option at Point A =

$$\frac{[\text{Value of option at Point B} \times \text{P}] + [\text{Value of option at Point C } \times (1-\text{P})]}{1+\frac{\text{R}}{100}}$$

Note: Sometimes Question requires us to calculate Expected rate of return on option:

Formula is = $\frac{\text{Value of Option at Maturity} - \text{Option Premium Paid in beginning}}{\text{Option Premium Paid in beginning}}$ * 100

Delta Hedging

Delta: Change in value of option due to change in price of share

For example: Maturity period is 3 months, CMP of share is 100. Call option at strike price of 150 has premium of 10, which means value of call option is 10

Let us say after one month price of share goes up to 110 and value of option goes up to 15.

Increase in share price = 110 – 100 = 10
Increase in value of option = 15 – 10 = 5

At the end of 2 months price of share goes upto 120 and value of premium goes upto 20
Increase in share price = 120 – 110 = 10
Increase in value of option = 20 – 15 = 5

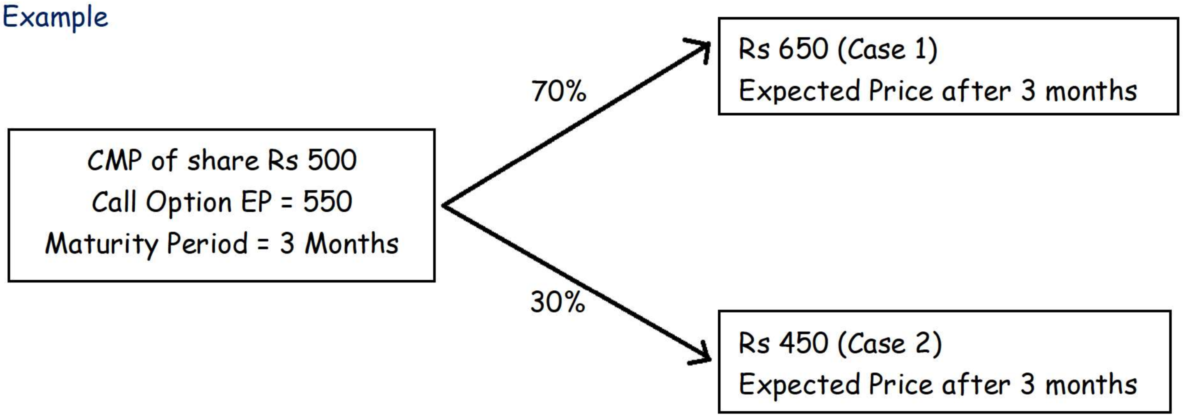
Therefore, value of option goes up by Rs 5, when price of share increases by 10

[This relationship/ ratio is called as Delta] Ratio= $\frac{15-10}{110-100}$= $\frac{5}{10}$ = 0.5

Formula to calculate Delta is therefore = $\frac{\text{Change in Value of Option}}{\text{Change in Value of Price of Stock}}$

To Do Delta hedging, write 1 call option and Buy share in the ratio calculated above
In our example above, Hedging will be done by writing/ selling one option and Buying 0.5 share

Example



Value of Option at Maturity
Case 1: 650 – 550 = 100
Case 2: Option Lapse = 0

Therefore, change in value of option = 100 – 0 = 100
Change in price of stock after 3 months = 650 – 450 = 200

Delta Ratio = $\frac{100}{200} = 0.5$
Delta Hedging = Write one call option and Buy 0.5 share

Analysis of Position after 3 months

Price goes up to 650		Price goes down to 450	
Loss on Call Option	-100	Call Option lapse	0
Sale of 0.5 share (650 x 0.5)	325	Sale of share (450 x 0.5)	225
	225		225

So we can see from above, after delta hedging, net pay off remains same in both the situations

Black Schole Model (BSM)

Value of Call Option = $(\text{CMP} \times \text{Nd1}) - \left(\frac{\text{EP}}{e^{rt}} \times \text{Nd2}\right)$

Value of Put Option = $\frac{\text{EP}}{e^{rt}} - \text{CMP} + \text{Value of Call Option}$

$$d1 = \frac{\ln\frac{\text{CMP}}{\text{EP}} + (r + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$$

$$d2 = d1 - \sigma\sqrt{t}$$

$CMP = Current\ Market\ Price$
$EP = Exercise\ Price$
$r = Risk\ free\ rate\ of\ Return$
$\sigma\sqrt{t} = Periodic\ standard\ Deviation$

Note: Nd1 and Nd2 = Cumulative area in normal distribution table

Note:
1. If expected dividend is given, then present value of dividend is reduced from current market price.
2. If dividend rate is given, then deduct dividend rate from r in all the formulas used above

Logic of deducting dividend from spot price is, after declaration of dividend, Market price becomes cum market Dividend price which means dividend component gets included in it. So we need to deduct dividend component to get actual market price

Note steps to calculate value of e^{rt} and $\ln\frac{\text{CMP}}{\text{EP}}$

Example 1: Rate of interest is 12% pa continuously compounding: $e^{\frac{12}{100}\times 1}$ = 1.1275

Standard steps:

0.12
÷ 4096
+1
X= 12 times

Example 2: Calculate value of $\ln 1.1275$ = 12%

Standard steps:

$\sqrt{12}\ Times$
- 1
× 4096

Futures Contract

Forward Contract: Agreement between 2 parties to purchase/ sale particular item at pre-determined price at pre-determined date. Both parties have to fulfil contract i.e. they have obligation under contract.

FORWARD CONTRACT	FUTURE CONTRACT
Over the counter	Traded on stock exchange
No Margin required	Margin required
Settlement on Maturity only	Daily Mark to Margin

Future Contract Pricing (Cost of carry Model)

RG expects price of reliance share after 3 months to be 600.
CMP of RIL share = 500
Stock exchange is quoting future price of RIL = 515
Borrowing rate from bank = 10% pa

RG purchases RIL share at CMP by borrowing 500 from Bank. After 3 months RG will pay to Bank = $500\left(1 + \frac{10\times\frac{3}{12}}{100}\right)$ = 512.5

At maturity, sold RIL share at 600 and paid 512.5 to Bank. Profit = 600 - 512.5 = 87.5

So RG will compare 512.5 and 515 to decide whether future contract to be purchased or not

Let's say in above example, Dividend of 2.5 is expected after 3 months. Dividend will be received only when share is purchased. In that case, Effective cost when Ram borrowed = 512.5 - 2.5 = 510
Cost quoted by stock exchange = 515

Arbitrage Process:

Example CMP = 500, 6 month future price = 542, Rate of Interest = 10% pa

Therefore Future Price = $500\left(1+\frac{10\times\frac{6}{12}}{100}\right)$= 525

Action:
Borrow 500 and Buy share of 500 (BUY SPOT)
Enter into contract to sell share after 6 months at 542 (SELL FUTURE)

At maturity:
Sell the share at 542 and receive 542
Pay off bank loan along with Interest i.e. 525
Net gain = 542-525= 17

Arbitrage process summary:
When future price is overpriced: Buy spot and Sell Future
When future price is under-priced: Sell spot and Buy Future

Theoretical Future price = Spot + Interest – Dividend

In case of simple Interest = Spot$\left(1+\frac{R\times\frac{t}{12/365}}{100}\right)$ where t means number of days/ months

In case of monthly compounding Interest = Spot$\left(1+\frac{\frac{R}{12}}{100}\right)^{t}$ where t means number of months

In case of continuously compounding Interest =($Spot\ x\ e^{R\times\frac{t}{12/365}}$) where t means number of days/ months

In all the three cases above, R means rate of Interest p.a.

As seen from above example, if dividend is given in question, dividend is to be deducted to calculate theoretical future price

If Fixed amount of dividend is given in question, then we can do either of the following:
(Spot + Interest) – Dividend OR
(Spot – PV of Dividend) + Interest [Followed by ICAI]

If Dividend is given in percentage form, then to calculate Future price:
Deduct Dividend percentage from rate of interest in above 3 formulas to get future price

Note: Basis = SPOT – FUTURE price
When SPOT price > Future price = Backwardation Market
SPOT price < Future price = Contango

Beta Management

Beta= Sensitivity of Return with respect to market

$$\beta = \frac{\text{Change in Stock Return}}{\text{Change in Market Return}}$$

Beta Management means adjusting beta of portfolio on basis of market expectations
Beta should be higher, when market is expected to rise
Beta should be lower, when market is expected to Fall

How to hedge using future contracts
Suppose Rohan has shares of 500,000 of RG Ltd. Share of RG ltd has a beta of 2, which means if market moves by 10%, share of RG Ltd will move by 10x2= 20%

Rohan expects that market is expected to fall by 10%

Consequences
RG Ltd share will go down by 10x2= 20%, therefore, loss on shares will be 500000x20%= 100,000.
Now Rohan will short NIFTY Futures of such an amount that gain of NIFTY Futures is 100,000

$$(X * 10\% = 100{,}000)$$
$$X = 10{,}00{,}000$$

Rohan will therefore short NIFTY Futures of Rs 10,00,000 i.e.
= Value of Portfolio × Beta of Portfolio
= 5,00,000 x 2
= 10,00,000

Generally futures are traded in LOTS. Let the price of NIFTY is 200 and Lot size is 50
So to calculate number of lots, Divide the total amount of NIFTY Futures with the price of NIFTY, we will get number of NIFTY Futures to be purchased
Then divide it by LOT size, we will get number of LOTS
i.e. $\frac{10{,}00{,}000}{200} = 5000$

Therefore 500 NIFTY is to be purchased. Contract size is 50
So Number of Lots is 5000/50 = 100 Lots

So Final Formula to calculate number of Lots is = $\frac{\text{Value of Portfolio} \times \text{Beta}}{\text{Price of NIFTY} \times \text{Lot Size}}$

Important Note: If number of contracts comes in decimal, round it off to upper whole number
Above formula is used when investor wants to perfectly hedge its portfolio

When perfect hedge is not required, then below formula is used:

$$\frac{\text{Value of Portfolio } \times (\text{Beta} - \text{Required Beta})}{\text{Price of NIFTY } \times \text{Lot Size}}$$

Suppose in above eg, Rohan wants that his portfolio instead of having beta 2, should have beta 1.5 which means he wants portfolio to be impacted by market changes to extent of 1.5 only, in that case, Rohan will enter into Future contract as below:

$$\frac{(500{,}000\ (2 - 1.5)}{200 \times 50} = 25 \text{ Contracts}$$

Summary of Hedging using Future contract

If we are long on any share i.e. Holding share = Then we will short Futures to Hedge
If we are short on any share i.e. Short sell= Then we will Buy Futures to Hedge

Margin Requirement in Future Contract

Initial margin = Amount initially deposited with stock exchange on entering into contract
If % of initial margin is given in question, use it to calculate margin amount = $\text{Contract Value} \times \%\text{ given}$

Otherwise use formula = $\mu + 3\sigma$
μ = Average change in contract value
σ = Standard Deviation

Mark to Market Margin= Gain / loss on daily settlement on future contract
In case of gain = Added to Initial margin amount
In case of loss= Deducted from Initial margin amount

Calculation of M2M=
$\text{Number of Lots} \times \text{Lot size} \times (\text{Closing price of Future today} - \text{Closing price of Previous day})$

Maintenance Margin: When balance in initial margin account goes below this maintenance margin limit, Investor deposits more money to restore its Initial margin limit

Commodity Derivatives= Derivative contracts in which the underlying asset is a commodity like gold, silver, rice etc

Hedging using commodity Futures: (Exactly same as future contracts studies above)
If we have long position on commodity = SHORT FUTURES
If we have short position on commodity = BUY FUTURES

Number of Future contracts to be bought/ sold = $\frac{\text{Value of Commodity} \times \text{Beta}}{\text{Price of Commodity future} \times \text{Lot Size}}$

In question, if Beta is not given, then we will calculate Beta (hedge Ratio) = $\frac{\sigma_s}{\sigma_f} \times r_{sf}$

σ_s= Standard deviation of change in spot price of commodity
σ_f= Standard deviation of change in future price of commodity
r_{sf}= Correlation between change in spot and Future price`

Real Options:
Call Option: Investment Option
Put Option: Abandonment Option

Value of real option is to be calculated same as studied in Binomial model (Risk neutral probability method) Formula of risk neutral probability was = $\frac{R-d}{\mu-d}$

However in real option, risk neutral probability is to be calculated on basis of return to get marks in exam

Real option Example:
Cost of project = 500, Cost of capital 10%, Calculate NPV
Cash Inflow: Year 1 = 300, Year 2 = 150, Year 3 = 100

Present value of cash flows = $\frac{300}{1.1} + \frac{150}{(1.1)^2} + \frac{100}{(1.1)^3} = 471.826$

NPV = 471.826 - 500 = -28.174

-500 Today	300 Year 1	150 Year 2	100 Year 3 Investment =1000	70% 30%	Cash Inflow 120 pa forever PVCI = 1200 Cash Inflow 80 pa forever PVCI = 800

At year 3 end, If favourable govt comes, cash inflows will be 120 pa forever, otherwise it will be 80L pa
NPV at end of year 3
If Favourable govt comes = $\frac{120}{10\%} - 1000 = 200 \text{ Lakh}$
If Favourable govt does not comes = $\frac{80}{10\%} - 1000 = -200 \text{ Lakh}$

At year 3 end, you have following 2 options,
Invest 1000 L and earn 200L with probability of 70% OR
Invest nothing since NPV is negative in this case

Therefore Value of call Option = $\frac{(200\times0.7)+(0\times0.3)}{(1.10)^3}$ = 105.184
Net NPV = 105.184 – 28.174 = 77.01

Note: While calculating risk neutral probability on the basis of return for real option, dividend can also be given in question. In that case, dividend will also be considered for calculating probability

Timing Option= Whether to do investment today or wait some time to do investment

BSM in real option: While calculation of value of option we studied that if dividend amount is given, we adjust CMP to deduct dividend component from it, since we assume CMP is cum dividend

When dividend is given in amount terms then we deduct present value of dividend from CMP
But where dividend is given in yield terms, then adjusted CMP is to be calculated by using below formula
= $X\left(1 + \frac{\text{Dividend Yield Rate}}{100}\right) = \text{CMP}$ where X will be our adjusted Current Market Price

So Formula to calculated Adjusted Market price is = $\frac{\text{CMP}}{1+\frac{\text{R}}{100}}$ or $\frac{\text{CMP}}{\text{e}^{\text{rt}}}$